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Readers' Forum

Brief discussion of previous investigations in the aerospace sciences and technical comments on papers published in the AIAA Journal *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

Comment on "Conservation Errors and Convergence Characteristics of Iterative Space-Marching Algorithms"

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IN Ref. 1, the authors have critically examined the new iterative space-marching algorithms for numerical solution of the parabolized Navier-Stokes equations. They claim to have shown by analysis and numerical examples that employing parabolized flux difference splitting (PFDS) in the marching direction can lead to serious conservation errors even though a finite volume formulation is employed. The authors state that the parabolized flux vector splitting (PFVS) is conservative in the telescoping sense and that the quantity that is conserved depends on the form of splitting that is employed. They also state that the PFDS scheme uses different values for the left and right face, and, hence, it is not possible to develop a PFDS scheme that is conservative. They seem to conclude that the PFVS scheme with Vigneron splitting[1] is better than the PFDS scheme since the former is conservative.

In a Vigneron type of flux vector splitting, only the pressure gradient term is split into a downstream and an upstream component, and when the upstream components of the fluxes are neglected in subsonic and/or reverse flow regions, the mass flux term is retained in its full form. Hence, one does not have to perform any computation to conclude that the mass flux conservation is achieved to a great degree of accuracy when such a flux vector splitting is employed.

In the case of the hypersonic inlet problem, the authors have presented figures to show only the error in the conservation of mass flux. Since a good estimate of the accuracy of a solution can be obtained only when the errors in all of the conserved quantities are determined, it appears that one cannot draw any conclusion regarding the accuracy of the PFVS method with Vigneron splitting by just looking at the conservation error in only one of the fluxes, especially the mass flux, which, due to the type of splitting employed, should be conserved exactly. All the authors have succeeded in demonstrating is that they probably do not have any coding errors.

Accumulated error depends on the streamwise spacing employed and the overall grid resolution in the marching plane. When the same problem was studied using the USA-RG2D code, two to three orders of magnitude less error in the mass flux was encountered. A time-marched solution for the same problem indicated a fairly large region of separated flow, revealing that the space-marched solution with or without accurate conservation of mass is likely to be inaccurate.

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Both the PFVS and the PFDS schemes when applied to subsonic flow problems introduce errors in the solution. For attached flows, in the PFVS scheme with Vigneron splitting, the error arises from neglecting the upstream component of the pressure gradient term in the x-momentum equation, whereas in the case of the PFDS scheme of Ota et al.,[2] it stems from neglecting the u-c component of all of the fluxes. In the limiting case of Mach number $\rightarrow 0$, the former completely neglects the pressure gradient term, whereas the latter neglects only a part of it. Such being the nature of these two approximations, is it possible to conclude that one method is better than the other just because it conserves one of the four quantities (or five in three dimensions) exactly, even though its conservation error in the other quantities may be larger? Is it possible to conclude that a scheme that conserves mass exactly and introduces larger errors in x momentum leads to a more accurate solution than a scheme that introduces smaller error in all of the conserved variables? Also, the nature of the error depends on the problem under consideration, and, hence, it is not possible to arrive at a definite conclusion about the methods based on just one or two problems.

References

[1]Thompson, D. S., and Matus, R. J., "Conservation Errors and Convergence Characteristics of Iterative Space-Marching Algorithms," *AIAA Journal*, Vol. 29, No. 2, 1991, pp. 227-234.

[2]Ota, D. K., Chakravarthy, S. R., and Darling, J. C., "An Equilibrium Air Navier-Stokes Code for Hypersonic Flow," AIAA Paper 88-0419, Jan. 1988.

Reply by Authors to S. V. Ramakrishnan, D. K. Ota, and S. R. Chakravarthy

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THE criticism of Thompson and Matus[1] by Ramakrishnan et al.[2] is quite interesting since none of the basic findings of Ref. 1 are disputed. The criticism appears to be based on the perception that the authors concluded that parabolized flux difference splitting (FDS) was inferior to parabolized flux vector splitting (FVS). This conclusion was never explicitly stated in Ref. 1. In fact, the question of solution accuracy is a

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topic that was not addressed in the paper. Any parabolized Navier-Stokes (PNS) algorithm is subject to error purely because of the single-pass philosophy. A second source of error is the modification of the flux vector that occurs to allow the space-marching solution to be obtained. Both the parabolized FDS and FVS algorithms are subject to these errors. However, the simple fact of the matter is that the parabolized FDS algorithm is not conservative in subsonic regions of the flowfield. This is what was demonstrated, among other points, in the paper in question. By construction, the parabolized FVS algorithm exactly conserves the mass flux upon convergence of the iteration at a given marching station. The mass flux was chosen as the quantity to monitor since the other momentum and energy equations effectively have source terms, i.e., the viscous terms. However, the remaining quantities in the modified flux vector are also exactly conserved when the effects of the viscous terms are taken into account. The fact that the mass flux error occurring when parabolized FDS is used is grid dependent, as correctly noted in the Comment, certainly does little to mitigate the consequences of its presence.

In Ref. 1, a numerical integration of the fluxes used in the solution algorithm along an ξ = const line was used to compute the mass flux at each marching station. The error in mass flux was defined as the computed mass flux minus the exact mass flux divided by the exact mass flux. The errors were displayed as a percentage of the total mass flux at each marching station. In results not included in Ref. 1, doubling and halving the grid spacing in each direction produced a maximum local change in the mass flux error of less than a factor of 3. However, according to Ramakrishnan et al.,[2] the mass flux errors computed using a similar computer code[3] were "2 to 3 orders of magnitude less" than the mass flux errors shown in Figs. 2 and 4 of Ref. 1. Therefore, it seems unlikely that grid sensitivity alone is responsible for the discrepancy in reported mass flux errors. Since there are many possible interpretations of a mass flux error and since Ramakrishnan et al.[2] do not describe their method of determining the mass flux error, we can only attribute the discrepancy in reported values to their use of a different error measure.

Although not considered in the paper, several points raised in the Comment concerning the relative accuracy of the two methods merit brief discussion. The statement in the Comment that neglecting the $u - c$ eigenvalue eliminates only a part of the streamwise pressure gradient in the limiting case of $M \rightarrow 0$ is misleading. The effect of this term, in conjunction with the associated right eigenvector, is to modify each of the terms in the interface flux, not just the pressure term. It is in no way apparent that the net effect of neglecting the flux increment associated with this eigenvalue coupled with a nonconservative formulation is superior to the Vigneron approach in a conservative formulation as conjectured in the Comment. To our knowledge, this question has never been addressed.

Incidentally, the character of the flowfield associated with the hypersonic inlet, particularly the streamwise separation, is well known and has been previously reported.[4] This case was included because it had been computed by other researchers and, presumably, was familiar to the PNS community.

References

[1]Thompson, D. S., and Matus, R. J., "Conservation Errors and Convergence Characteristics of Iterative Space-Marching Algorithms," *AIAA Journal*, Vol. 29, No. 2, 1991, pp. 227–234.

[2]Ramakrishnan, S. V., Ota, D. K., and Chakravarthy, S. R., "Comment on 'Conservation Errors and Convergence Characteristics of Iterative Space-Marching Algorithms,'" *AIAA Journal*, Vol. 30, No. 4, 1992.

[3]Ota, D. K., Chakravarthy, S. R., and Darling, J. C., "An Equilibrium Air Navier-Stokes Code for Hypersonic Flow," AIAA Paper 88-0419, Jan. 1988.

[4]Newsome, R. W., Walters, R. W., and Thomas, J. L., "An Efficient Iteration Strategy for Upwind/Relaxation Solutions to the Thin-Layer Navier-Stokes Equations," AIAA Paper 87-1113, June 1987.

Errata

Critical Evaluation of Two-Equation Models for Near-Wall Turbulence

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DURING typesetting of this article, several errors were introduced that critically affect its content. Corrected reprints are available from the authors.

Page 325:

The last term in Eq. (9) should be preceded by a minus sign and is missing an overbar:

$$\mathcal{P}_\epsilon = -2\nu\,\overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial u_k'}{\partial x_j}}\,\frac{\partial \bar{u}_i}{\partial x_k} - 2\nu\,\overline{\frac{\partial u_j'}{\partial x_i}\frac{\partial u_j'}{\partial x_k}}\,\frac{\partial \bar{u}_i}{\partial x_k} - 2\nu\,\overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_i'}{\partial x_j}\frac{\partial u_k'}{\partial x_j}} - 2\nu\,\overline{u_k'\frac{\partial u_i'}{\partial x_j}}\,\frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_k} \tag{9}$$

Page 326:

The third minus sign was omitted in Eq. (32):

$$\frac{D\tau}{Dt} = \frac{\tau}{K}\tau_{ij}\frac{\partial \bar{u}_i}{\partial x_j} - 1 - \frac{\tau}{K}\mathfrak{D} - \frac{\tau^2}{K}\mathcal{P}_\epsilon + \frac{\tau^2}{K}\Phi_\epsilon + \frac{\tau^2}{K}\mathfrak{D}_\epsilon + \frac{2\nu}{K}\frac{\partial K}{\partial x_i}\frac{\partial \tau}{\partial x_i} - \frac{2\nu}{\tau}\frac{\partial \tau}{\partial x_i}\frac{\partial \tau}{\partial x_i} + \nu\nabla^2\tau \tag{32}$$

The sentence following Eq. (35) should end with "($\ldots C_\mu = 0.09$)."

Page 327:

In Eq. (35), the last variable in the partial derivative should be changed from "x" to "x_i."